7.2 – Orthogonal Diagonalization

Definition: If A and B are square matrices, then we say that B is **orthogonally similar** to A (or that A and B are **orthogonally similar matrices**) if there is an orthogonal matrix P such that $B = P^T AP$.

If A is orthogonally similar to some diagonal matrix, say $P^T AP = D$, then we say A is **orthogonally diagonalizable** and that P **orthogonally diagonalizes** A.

Theorem 7.2.1 If A is an $n \times n$ matrix with real entries, then the following are equivalent.

a) A is orthogonally diagonalizable.

b) A has an orthonormal set of n eigenvectors.

c) A is symmetric.

#2 Find the characteristic equation of the given symmetric matrix, and then by inspection determine the dimensions of the eigenspaces.

$$\begin{bmatrix} 1 & -4 & 2 \\ -4 & 1 & -2 \\ 2 & -2 & -2 \end{bmatrix}$$

Theorem 7.2.2 If A is a symmetric matrix with real entries, then:
a) The eigenvalues of A are all real numbers.
b) Eigenvectors from different eigenspaces are orthogonal.

#11 Find a matrix P that orthogonally diagonalizes A, and determine $P^{-1}AP$.

$$A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$